The University of Georgia
Mathematics Education
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Bouncing Barney

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The idea of Bouncing Barney is that he is in a triangular enclosure and beginning from some point in the plane, he travels in a direction parallel to one side of the triangular enclosure until he reaches one of the boundaries or its extension. At that point Barney then changes direction and travels parallel to another side of the enclosure. There are multiple investigations to explore the path Barney may take, looking not only at path alternatives, but also the distances.

Prove that Barney will eventually return to his starting point.
How many times will Barney reach a wall before returning to his starting point?

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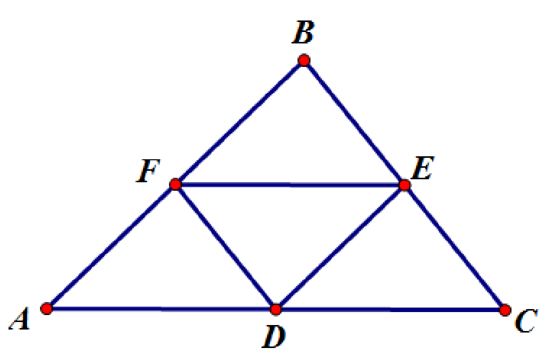
Let's investigate first by starting at the midpoint of AC. Barney will travel towards AB parallel to BC. Ones it intercepts with AB it will travel towards BC parallel to AC. Once it intersects BC it will travel towards AC parallel to AB. Barney ended up at his starting point after only three steps.



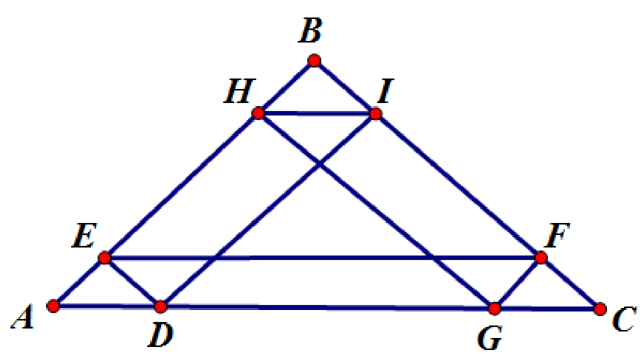

Why only three steps?

We can first look at similar triangles. Triangle AFD is similar to triangle ABC. First, both triangles share <BAD. Next, it is given in the path of barney that DF and BC are parallel, and are both cut by transversal EF and AC, meaning the corresponding angles <AFD and <ABC, and <ADF and <ACB, are congruent.

In addition, we know if a line is parallel to one side of a triangle and intersects the other two sides of the triangle, then it divides each of the two sides into proportional segment lengths. Since we know that DF is parallel to BC, we know that AF=AD. The same logic can be applied throughout the rest of the triangle, so that BC=BE and EC=CD.

So since we began at the midpoint of AC at point D, and DF is parallel to BC, then F must be the midpoint of BA. Therefore, Barney will touch each side of the triangle at each midpoint, returning to its original starting point after three steps.

Now let's look at Barney when he starts at a random spot on the triangle. Here he starts at point D. Barney will travel parallel to BC until he reaches AB at point E. He then travels parallel to AL until he reaches BC at point F…..etc. It takes Barney 6 steps to return to his starting point. No matter what point I chose to start at on triangle ABC, Barney returned to his starting point in 6 steps, unless he started on a midpoint or on point A, B, or C, which resulted in three steps.




There are several things we can deduct by looking at this triangle. First, due to parallel lines being cut by a transversal (as explained in the previous example), we can conclude that triangle BHI is similar to triangle BEF. Angle B is congruent to angle B. <BHI and <BEF are congruent because they are both corresponding angles of parallel lines. The same logic can be applied to <BIH and <BFE.

From this we know that triangle BHI and triangle BEF are similar, and therefore have proportional corresponding sides. We can also use the same logic to show that BHI, BEF, and BAC are all similar triangles.

From our previous triangle, we can show that triangle EAD and BAC are also similar triangles. <A is congruent to <A. <AED and <HBI are congruent because they are both corresponding angles of parallel lines. The same logic can be applied to <AED and <HIB. This logic can also be applied to other triangles such as triangle BHI and FGC.

Let's see if we can add some values to make a conjecture about the distance that Barney travels.

We know Barney starts at point D and travels parallel to BC until he intersects AB at point E. Let's say he started at point D, which is 1/x of AC. For example purposes let's say that AD is 1/6 of AC. Because Barney travels parallel to BC, the line segment ED will cut the other two sides of the triangle into equal proportional segments. Therefore, AE will also be 1/6 of AB. We have shown that triangle BHI is congruent to AED, therefore HB will also be 1/6 of AB.

Next, we have also shown that tringle BHI is similar to triangle BEF. We can say that BH is (1/6)/(5/6)=1/5 of BE. The sides of the triangles must be proportional because they are similar, so BI is 1/5 of BF and HI is 1/5 of EF. We can also apply the same logic to triangles BHI and BAC because they are also similar triangles. HI is 1/4 of AC. We concluded that BE is ¾ of AB, therefore EF will also be ¾ of AC. Since EF is ¾ of AC and HI is ¼ of AC, adding these together we get AC.

We can apply similar logic throughout the triangle.

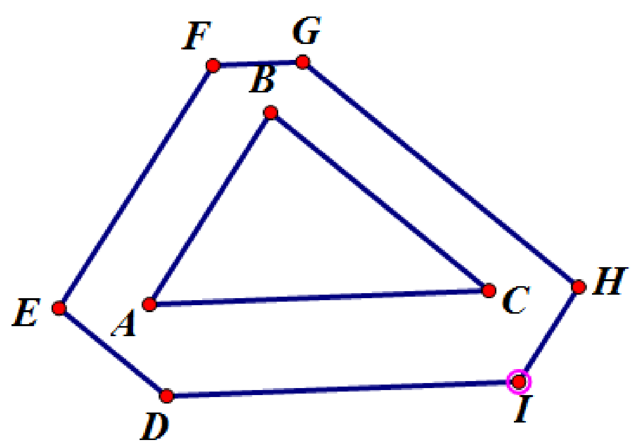
We can conclude that the path Barney takes has the same perimeter as the perimeter of triangle ABC.

HI+EF+ED+HG+FG+ID=AC+BC+AB.

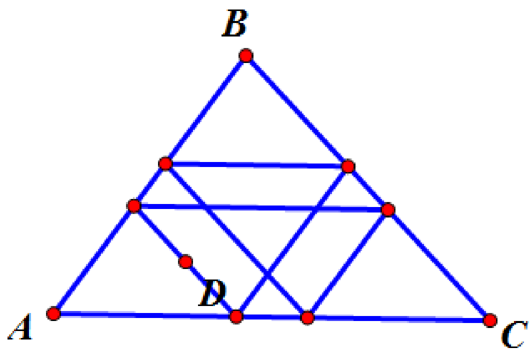
To wrap up: A line parallel to one side of a triangle that intersects the other two sides of the triangle will intersect these lines to create segments of a proportional relationship. Because Barney must travel in a manner in which he is parallel to one side of the triangle, this relationship will continue as he moves throughout the triangle. As long as he starts inside or on the triangle at a point that is not the midpoint of one of the sides of the triangle, he will travel the same perimeter as the triangles perimeter which will lead him back to his initial point.

We can see these measurements in GSP.

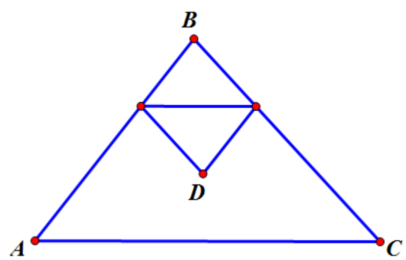
What happens when Barney starts at a point outside of the triangle? It still takes Barney 6 steps to return to his beginning spot.




What happens when Barney starts at a random point inside the triangle? It still takes Barney 6 steps to return to his beginning spot and he reaches a wall 6 times.




What if we start at the centroid of the triangle?




It takes Barney three steps to return to his starting point and he only reaches two walls. We can also measure the distance it takes him to return to his starting point which is 1/3 of the perimeter of the triangle. This path is similar to the path he takes when starting at a midpoint.